Little Theories in Six and Seven Dimensions

Riccardo Argurio[1] and Laurent Houart[2]

Service de Physique Théorique
Université Libre de Bruxelles, Campus Plaine, C.P.225
Boulevard du Triomphe, B-1050 Bruxelles, Belgium

Abstract

We discuss theories with 16 and 8 supercharges in 6 and 7 dimensions. These theories are defined as world-volume theories of 5- and 6-branes of type II and M theories, in the limit in which bulk modes decouple. We analyze in detail the spectrum of BPS extended objects of these theories, and show that the 6 dimensional ones can be interpreted as little (non-critical) string theories. The little 5-branes of the 6 dimensional theories with 16 supercharges are used to find new string theories with 8 supercharges, which have additional group structure. We describe the web of dualities relating all these theories. We show that the theories with 16 supercharges can be used for a Matrix description of M-theory on T^6 in the general case, and that they also reproduce Matrix theory on T^5 and T^4 in some particular limit.

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[1] Aspirant F.N.R.S. (Belgium). E-mail: rargurio@ulb.ac.be
[2] Chercheur I.I.S.N. (Belgium). E-mail: lhouart@ulb.ac.be

1   Introduction

A promising approach to the understanding of M-theory [1, 2, 3] is the so-called M(atrix) theory [4]. According to this original proposal, the supersymmetric $U(N)$ matrix quantum mechanics of N D0-branes describes M-theory in flat 11 dimensional space, in the infinite momentum frame (IMF) when $N \to \infty$ or alternatively in the discrete light cone quantization [5] for finite N. M-theory toroidal compactifications are described by an equivalent M(atrix) model in which the matrix quantum mechanics is replaced by super Yang-Mills (SYM) [4, 6, 7, 8] on a dual torus. However when there are more than three compact dimensions the SYM is ill-defined because it is non-renormalizable (see e.g.[9]). In order to circumvent this problem one has to go beyond the SYM prescription. Matrix theory on T^4 is described in terms of a $(2, 0)$ field theory in 5+1 dimensions [10, 11], which corresponds to the theory on the world-volume of N coinciding M5-branes [12]. Compactifying further one has to abandon the idea of having a field theory description. On T^5, Matrix theory is believed to be described in terms of a non-critical string theory in 5+1 dimensions [11, 13] obtained from N NS 5-branes at vanishing type II string coupling. On T^6, a description using the M-theory KK 6 monopole has been recently proposed [14, 15]. It appears that this 6+1 dimensional theory contains membranes, and it has been called "m-theory" [16].

The similarity between m-theory and M-theory is actually striking. One can indeed define "little string theories" in 5+1 dimensions, one chiral and one non-chiral, and relate them to m-theory by T-dualities and (de)compactification. These theories can be defined using 5-branes of several types appearing in type II and M theories[1], always with additional transverse compact directions [16]. It has to be stressed that these additional compact directions introduce new parameters with respect to the theories defined by Seiberg [13], thus making them suitable for a description of Matrix theory on T^6.

In this paper, we find interesting to study the little string theories and m-theory in their own respect. We first revisit the theories in 6 and 7 dimensions with 16 supercharges leading to iia, iib little string theories and m-theory. The different ways to obtain these theories are analyzed. We start from 5 and 6 dimensional extended objects defined in M or type II theories and we take limits in which bulk modes decouple. This leads nevertheless to a non-trivial theory without gravity defined on the world-volume of the extended objects. We show the web of dualities between these little theories which exactly reproduces the scheme of the "big" theories in 10 and 11 dimensions. In a Matrix theory perspective, the spectrum of the BPS extended objects of these little theories is investigated and it is shown that it agrees with the U-duality group of M-theory compactified on T^6. Furthermore the theories used to describe M-theory on T^4 and T^5 are recovered as particular limits of these little string theories.

We then turn to theories in 6 dimensions with 8 supercharges. These theories have $(1,0)$ supersymmetry, do not contain gravity and may have an additional gauge symmetry. Our strategy is to obtain them from the theories with 16 supercharges. We mimic the 10 dimensional procedure in which type I theory is obtained from IIB theory introducing an $\Omega 9$ orientifold and 16 D9-branes [18, 19, 20] (see also [21]). The two heterotic string

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[1] This approach was pioneered in the work of [17].

theories are then found by chains of dualities. Applying the same procedure to the 6 dimensional theories, we find one theory with open strings and two with closed strings, which we call respectively type i, h_b and h_a theories. These are in fact classes of theories. Unlike the 10 dimensional case, the gauge group is not constrained. Moreover, there is no simple description of the gauge theory defined by the h_a "little heterotic" theories, since this is again related to the (2,0) theory. As a consistency check of the picture, the h_a theory can also be related to a particular compactification of m-theory.

The paper is organized as follows. In section 2 we study the theories with 16 super-charges. For each one of the three theories we review the different ways to obtain them and explain how the limits taken are related by a chain of dualities. The limits take a particular simple form in the formulation which uses the KK monopoles. The relation between the little theories and compactifications of Matrix theory on tori is explained. In section 3, we consider the theories with 8 supercharges. We propose the definition of little type i open string theory, and of two kinds of little "heterotic" theories. The final section contains a brief discussion.

2   Theories with 16 supercharges

Supersymmetric theories with 16 supercharges naturally appear in type II string theories and M-theory as the effective theory on the world-volume of BPS branes. In order to have well-defined theories on world-volumes one has to take a limit in which the bulk modes decouple. This is achieved by sending the Planck mass, defined with respect to the non-compact space, to infinity.

We will consider here theories defined on the world-volume of 5 branes and 6 branes, and such that at least three of the transverse directions are non-compact (in order to keep the space asymptotically flat). This allows for extra transverse compact directions, which will actually play a key rôle in defining the parameters of the little theories.

In M-theory, we have the following two objects:

- M5-brane, with up to 2 transverse compact directions parametrized by R_1 and R_2.

- KK6-brane, which has naturally a transverse compact direction, the so-called NUT direction (see [22] for a recent review on KK monopoles).

In type IIA theory we have the following three objects:

- NS5(A)-brane, with 1 transverse compact direction parametrized by its radius R_A.

- KK5(A)-brane, with its transverse NUT compact direction.

- D6-brane, with no compact transverse directions.

The objects we have in type IIB theory are:

- NS5(B)-brane, with 1 transverse compact direction.

- KK5(B)-brane, with its NUT compact direction.

- D5-brane, with 1 compact transverse direction.

All these branes are related by the usual dualities relating type II and M-theory. We will however distinguish between dualities which leave the world-volume of the branes unaffected, as transverse T-dualities for NS branes (NS5 and KK5), IIB S-duality and transverse compactifications, and dualities which on the other hand act on the world-volume, as T-dualities and compactifications along a world-volume direction of NS branes and T-dualities for D-branes.

Considering the dualities leaving the world-volume unaffected leads to three different families of branes each one defining one theory:

- iia: KK5(A) $\leftrightarrow$ NS5(B) $\leftrightarrow$ D5

- iib: KK5(B) $\leftrightarrow$ NS5(A) $\leftrightarrow$ M5

- m: KK6 $\leftrightarrow$ D6

These three theories are related by dualities which affect the world-volume of the branes. A T-duality along the world-volume of a NS5 or a KK5 changes from IIA to IIB and thus also from iia to iib. Compactification of the KK6 on one of its world-volume directions yields the KK5(A), thus relating iia and m theories via compactification. The same duality between little theories is obtained acting with a T-duality on the world-volume of the D6, which gives the D5. Note also that the D4-brane, which defines a theory in 5 dimensions, can be obtained either by a T-duality from the D5, or by compactification from the M5. This shows that once compactified, there is no longer difference between iia and iib theories in 5 dimensions.

Although the relations discussed above are rather formal at this stage, they exactly reproduce the same pattern of dualities of the 10 and 11 dimensional theories. We will show hereafter that in the proper limits in which the above little theories make sense (i.e. they decouple from the bulk), this structure still holds and aquires even more evidence.

We now turn to the description of the different little theories.

2.1 iia theory

As explained above, there are three ways to define type iia theory [16]. The six dimensional supersymmetry is (1,1). This is most easily found for the D5 brane from dimensional reduction of the $N = 1$ supersymmetry in $D = 10$ [23]. For the NS5(B) and the KK5(A) it has been discussed respectively in [24] and [22]. The type iia theory is thus non-chiral.

The first approach is based on the D5 with a transverse compact direction of radius R_B.

We look for all the objects which from the D5 world-volume point of view have a finite tension, i.e. we rule out branes extending in transverse non-compact directions. The relevant configurations of branes intersecting with the D5, and breaking further 1/2 of the supersymmetry are: D1$\subset$D5, F1$\mapsto$D5, D3$\mapsto$D5, NS5(B)$\mapsto$D5 and KK5(B)$\|$D5. The F1, D3 and NS5 have a boundary on the D5 [12, 25], and their only dimension transverse to it wraps around the transverse compact direction.

Generically, supergravity solutions preserving 1/4 of the supersymmetries and representing two intersecting branes can be computed [26, 27, 28, 29]. Their existence can be deduced by the compatibility of the two supersymmetry projections which characterize the configuration. The supersymmetry projections characterizing the branes are discussed in the appendix, where we also fix the notations.

Before taking the limit in which the bulk decouples, we have to fix the tension and the coupling of the little string theory on the world-volume of the D5-brane. Since we have three parameters at hand, namely the string length l_s, the string coupling of IIB theory g_B and the radius R_B, it will be possible to send the 9 dimensional Planck mass[2] to infinity while keeping a non-trivial little theory on the brane.

The only string-like object which lives on the D5-brane is the D1 string trapped to its world-volume [30]. We take it to define the fundamental little string of iia theory. Accordingly, its tension is defined by (using (31)):

$$t_a \equiv T_{D1} = \frac{1}{g_B l_s^2}. \tag{1}$$

The boundaries of the F1, D3 and NS5, which are respectively 0-, 2- and 4-dimensional closed objects, act as little "d-branes" for the $f1$ little string. Their tension is postulated to be inversely proportional to coupling of the little string theory g_a [16]. We have:

$$
\begin{array}{ccccccc}
t_{d0} & \equiv & T_{F1}R_B & = & \frac{R_B}{l_s^2} & \equiv & \frac{t_a^{1/2}}{g_a} \\
t_{d2} & \equiv & T_{D3}R_B & = & \frac{R_B}{g_B l_s^4} & \equiv & \frac{t_a^{3/2}}{g_a} \\
t_{d4} & \equiv & T_{NS5}R_B & = & \frac{R_B}{g_B^2 l_s^6} & \equiv & \frac{t_a^{5/2}}{g_a}
\end{array}
\tag{2}
$$

The above definitions are consistent and, taking into account (1) we have:

$$g_a = \frac{l_s}{g_B^{1/2} R_B}. \tag{3}$$

The last object to consider is the KK5, which actually fills the world-volume of the D5. We can nevertheless define its tension using (30):

$$t_{s5} \equiv T_{KK5} = \frac{R_B^2}{g_B^2 l_s^8} = \frac{t_a^3}{g_a^2}. \tag{4}$$

The $d4$ and $s5$ branes were overlooked in the analysis of [16], they are however defined by perfectly well-behaved 10 dimensional configurations. They are important in the identification of this little theory as a model for a toroidal compactification of Matrix theory as we discuss at the end of this section.

We have defined the string tension t_a and the string coupling g_a of the little theory. In order for this iia theory to make sense, we have to take a limit in which the bulk modes

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[2] We have to consider the Planck mass in 9 dimensions because one of the transverse directions is compact. Furthermore its radius will be sent to zero in the limit discussed above. Note also that this limit does not depend on the size of the directions longitudinal to the D5-brane. For simplicity, we take them to be infinite.

decouple i.e. a limit in which the nine dimensional Planck Mass M_p is going to infinity at fixed t_a and g_a. The Planck Mass is given by:

$$M_p^7 = \frac{R_B}{g_B^2 l_s^8} = \frac{g_B t_a^{7/2}}{g_a}.$$ (5)

The limit defining type iia is thus characterized by:

$$g_B \to \infty, \qquad l_s \to 0, \qquad R_B \to 0$$ (6)

We can also find the iia theory starting with the NS5(B)-brane with one transverse compact direction of radius $\tilde{R}_B$. We call, in this case, the string coupling of type IIB $\tilde{g}_B$ and the string length $\tilde{l}_s$. The 10 dimensional configurations breaking 1/4 supersymmetry which define the BPS objects living in 6 dimensions are simply obtained by S-duality from the ones discussed in the preceding approach. They are the following: F1⊂NS5(B), D1↦NS5(B), D3↦NS5(B), D5↦NS5(B) and KK5(B)‖NS5(B). The little iia string is identified to the fundamental string of type IIB theory. Its tension is simply given by: $t_a = \tilde{l}_s^{-2}$. The little string coupling is:

$$g_a = \frac{\tilde{g}_B \tilde{l}_s}{\tilde{R}_B}.$$ (7)

It can be obtained for instance computing $t_{d0} = T_{D1} \tilde{R}_B$. The limit in which the Planck mass goes to infinity is defined by $\tilde{g}_B \to 0$, $\tilde{R}_B \to 0$ and $\tilde{l}_s$ constant. This result is consistent with the S-duality transformations: $g_B \to \tilde{g}_B = 1/g_B, l_s^2 \to \tilde{l}_s^2 = g_B l_s^2$ and $\tilde{R}_B = R_B$ left unchanged.

The third object with a 6 dimensional (1,1) supersymmetric world-volume which can be used to define iia theory is the KK5 monopole of type IIA string theory, obtained by a T-duality on the transverse compact direction from the NS5(B)-brane. This direction becomes the NUT direction of the Euclidean Taub-NUT space transverse to the KK5 world-volume [22]. It appears that in this picture all the relevant configurations which preserve 1/4 supersymmetries are made up from branes of type IIA inside the world-volume of the KK5(A) [16]: FI⊂KK5(A), D0⊂KK5(A), D2⊂KK5(A), D4⊂KK5(A) and NS5(A)⊂KK5(A). This makes the identifiaction of t_a and g_a straightforward. The fundamental iia string coincides now with type IIA's F1, and thus $t_a = \tilde{l}_s^{-2}$. Since here also the "little" d-branes coincide with the D-branes of type IIA (with $p \leq 4$), also the little string coupling is given by the IIA one: $g_a = g_A$. It is easy to find by T-duality from the NS5(B) picture the limit in which the KK5 decouples from the bulk. Since under T-duality $g_B \to g_A = \frac{g_B \tilde{l}_s}{R_B}$, $R_B \to R_A = \frac{\tilde{l}_s^2}{R_B}$ and $\tilde{l}_s$ is unchanged, in the KK5(A) picture we have g_A constant and $R_A \equiv R_{\mathrm{NUT}} \to \infty$. The Riemann tensor of the Taub-NUT geometry vanishes in this limit, an indication that the KK monopole decouples from bulk physics.

We recapitulate the BPS spectrum of type iia theory in the following table. We list the different little branes and their mass considering now a compact world-volume

characterized by radii Σ_i with $i = 1 \ldots 5$ and volume $\tilde{V}_5 = \Sigma_1 \ldots \Sigma_5$. We include for later convenience the KK momenta w.

Brane	Mass
w	$\frac{1}{\Sigma_i}$
$f1$	$\Sigma_i t_a$
$d0$	$\frac{t_a^{1/2}}{g_a}$
$d2$	$\frac{\Sigma_i \Sigma_j t_a^{3/2}}{g_a}$
$d4$	$\frac{\tilde{V}_5 t_a^{5/2}}{\Sigma_i g_a}$
$s5$	$\frac{\tilde{V}_5 t_a^3}{g_a^2}$

Table 1: mass of the BPS objects in iia theory.

We also summarize below the different ways to obtain iia theory and the relation between the parameters.

	$D5$ $\frac{1}{g_B}, R_B, l_s \to 0$	$NS5(B)$ $\tilde{g}_B, \tilde{R}_B \to 0$	$KK5(A)$ $R_{NUT} \to \infty$
t_a	$\frac{1}{g_B l_s^2}$	$\frac{1}{l_s^2}$	$\frac{1}{l_s^2}$
g_a	$\frac{l_s}{g_B^{1/2} R_B}$	$\frac{\tilde{g}_B \tilde{l}_s}{\tilde{R}_B}$	g_A

Table 2: definitions of iia parameters.

2.2 iib theory

We recall that there are three approaches to this 6 dimensional theory, using respectively the M5-brane with two transverse compact directions, the NS5-brane of type IIA with one compact transverse direction and the KK5 monopole of type IIB [16]. These three different branes all have a world-volume theory with (2,0) chiral supersymmetry [24, 31, 12, 22].

The procedure by which we analyze the structure of iib little string theory is similar to the one described in the preceding subsection. We will however meet here an interesting structure of iib which is its s-duality. We begin with the M5 approach, where this duality is geometric.

The M5-brane set up is characterized by the 11 dimensional Planck length L_p and the two radii R_1 and R_2 of the two transverse compact directions. The configurations breaking 1/4 supersymmetry in M-theory leading to finite tension objects on the world-volume of the M5 are the following: M2$\mapsto$M5 with the M2 direction orthogonal to the M5 wrapping either R_1 or R_2; M5$\cap$M5=3; KK6$\supset$M5 with the NUT direction of the KK6 identified either with R_1 or R_2.

The boundaries of the M2-branes are strings on the M5, but we cannot immediately identify the fundamental iib little string because we have two different kinds of them. We simply choose one of the two (say, the boundary of the M2 wrapped on R_1) to be the fundamental and thus to have tension t_b, and the other to be the little $d1$ brane with tension $\frac{t_b}{g_b}$. This defines g_b. s-duality of iib is then simply the interchange in M-theory of R_1 and R_2 (this can actually be extended to a full $SL(2, Z)$ duality group considering M2-branes wrapped on (p, q) cycles of the torus). We have thus (cfr. (25)):

$$
\begin{aligned}
t_{f1} = & \ T_{M2}R_1 = \frac{R_1}{L_p^3} & \equiv t_b, \\
t_{d1} = & \ T_{M2}R_2 = \frac{R_2}{L_p^3} & \equiv \frac{t_b}{g_b}.
\end{aligned}
\tag{8}
$$

The little string coupling is then given by:

$$
g_b = \frac{R_1}{R_2}.
\tag{9}
$$

We can now identify the other world-volume objects by their tension:

$$
\begin{aligned}
T_{M5}R_1R_2 = & \ \frac{R_1R_2}{L_p^6} = \frac{t_b^2}{g_b} & \equiv t_{d3} \\
T_{KK6}R_2 = & \ \frac{R_1^2 R_2}{L_p^9} = \frac{t_b^3}{g_b} & \equiv t_{d5} \\
T_{KK6}R_1 = & \ \frac{R_1 R_2^2}{L_p^9} = \frac{t_b^3}{g_b^2} & \equiv t_{s5}
\end{aligned}
\tag{10}
$$

Note that under s-duality the $d3$ is inert and the $d5$ and $s5$ are exchanged.

We still have to find the limit in which the bulk physics decouples. Keeping t_b and g_b finite, the Planck mass in 9 dimensions is given by:

$$
M_p^7 = \frac{R_1 R_2}{L_p^9} = \left(\frac{t_b^2}{g_b}\right)\frac{1}{L_p^3},
$$

and goes to infinity when $L_p \to 0$. To keep the parameters of iib finite, we also have to take $R_1, R_2 \to 0$.

We now consider the NS5(A) approach. The parameters are the string length $\tilde{l}_s$, the string coupling $\tilde{g}_A$ of type IIA theory and the radius $\tilde{R}_A$ of the compact direction. The configurations, breaking 1/4 supersymmetry, leading to finite tension objects in the world-brane of the NS5(A) are: F1$\cap$NS5(A), D2$\mapsto$NS5(A), D4$\mapsto$NS(A), D6$\mapsto$NS(5) and KK5(A)$\|$NS5(A). In this framework the string tension t_b is defined by the fundamental string, namely $t_b = \tilde{l}_s^{-2}$. The little string coupling g_b is found by identifying the tension of the $d1$-brane from the configuration with the D2. We have:

$$
t_{d1} = T_{D2}\tilde{R}_A = \frac{\tilde{R}_A}{\tilde{g}_A\tilde{l}_s^3} \equiv \frac{t_b}{g_b}, \qquad\qquad g_b = \frac{\tilde{g}_A\tilde{l}_s}{\tilde{R}_A}
\tag{11}
$$

We obtain this picture from the previous one by dimensional reduction on R_1, $R_1 = \tilde{g}_A \tilde{l}_s$. The $\tilde{R}_A$ here is the previous R_2. In this case the limit is taken performing $\tilde{g}_A \to 0$ and $\tilde{R}_A \to 0$ at fixed t_b and g_b. Note that the s-duality in this picture is less straightforward to obtain from 10 dimensional string dualities (one has to operate a TST duality chain).

Turning now to the KK5(B) picture, we find that, as in the type iia case, the little string theory is the reduction to the world-volume of the KK5 of the physics of the objects that fit inside it. Thus we simply identify t_b with $\tilde{l}_s^{-2}$, g_b with g_B, s-duality with S-duality, $f1$ with F1 and so on. As in the previous KK5 case, the limit in which the bulk decouples involves taking the radius of the NUT direction to infinity.

We recapitulate the BPS spectrum of type iib theory in the following table. As for the iia case, we list the different little branes and their mass considering now a compact world-volume characterized by radii Σ_i with $i = 1 \ldots 5$ and volume $\tilde{V}_5 = \Sigma_1 \ldots \Sigma_5$.

Brane	Mass
w	$\frac{1}{\Sigma_i}$
$f1$	$\Sigma_i t_b$
$d1$	$\frac{\Sigma_i t_b}{g_b}$
$d3$	$\frac{\tilde{V}_5 t_a^2}{\Sigma_i \Sigma_j g_b}$
$d5$	$\frac{\tilde{V}_5 t_a^3}{g_b}$
$s5$	$\frac{\tilde{V}_5 t_a^3}{g_b^2}$

Table 3: mass of the BPS objects in iib theory.

We also summarize below the different ways to obtain iib theory and the relation between the parameters.

	$M5$ $L_p, R_1, R_2 \to 0$	$NS5(A)$ $\tilde{g}_A, \tilde{R}_A \to 0$	$KK5(B)$ $R_{NUT} \to \infty$
t_b	$\frac{R_1}{L_p^3}$	$\frac{1}{\tilde{l}_s^2}$	$\frac{1}{\tilde{l}_s^2}$
g_b	$\frac{R_1}{R_2}$	$\frac{\tilde{g}_A \tilde{l}_s}{\tilde{R}_A}$	g_B

Table 4: definitions of iib parameters.

As most easily seen in the pictures using the NS5 or the KK5 branes, there is a t-duality relating iia and iib little string theories. It is simply the 10 dimensional T-duality between IIA and IIB, applied on a direction longitudinal to the world-volume

of the above-mentioned branes. To be more specific, application of such a longitudinal T-duality maps, say, the NS5(A) picture of iib theory to the NS5(B) picture of iia theory, and similarly for the KK5 pictures. The behaviour of the BPS objects is the same as in type II string theories: KK momenta are exchanged with wound $f1$ strings, the $s5$ brane of one theory is mapped to the one of the other theory, and dp-branes become $d(p+1)$- or $d(p-1)$-branes for transverse or longitudinal t-dualities respectively. iia and iib theories are thus equivalent when reduced to 5 space-time dimensions or less.

2.3   m-theory

As stated at the beginning of this section, there are two objects with 7 dimensional world-volume in M/type II theories: the D6-brane in type IIA and the KK6 monopole in M-theory. The supersymmetry algebra is unique and obviously non-chiral.

We first consider the D6 approach. Note that for the transverse space to be asymptotically flat, we cannot have any compact transverse dimension. The free parameters are thus the string length l_s and type IIA string coupling g_A. Already at this stage we know that the theory on the world-volume will be characterized by only one parameter (one is lost taking the appropriate limit which decouples the bulk).

In this case, we have to consider configurations preserving 1/4 supersymmetries with a brane within the D6-brane. The only branes of type IIA for which this works are the D2- and the NS5-brane [29]. We identify them with the $m2$ and $m5$ branes. As it is necessary for the definition of m-theory, only one parameter suffices to define both their tensions. Indeed we have:

$$t_{m2} \equiv T_{D2} = \frac{1}{g_A l_s^3} \equiv \frac{1}{l_m^3}$$

$$t_{m5} \equiv T_{NS5} = \frac{1}{g_A^2 l_s^6} = \frac{1}{l_m^6}$$

(12)

l_m is thus the characteristic length of m-theory, the analog of the Planck length in M-theory.

In order to decouple gravity, we send the 10 dimensional Planck mass to infinity. Keeping l_m finite, we have:

$$M_p^8 = \frac{1}{g_A^2 l_s^8} = \frac{1}{(l_m^6)l_s^2},$$

and thus we have to take $l_s \to 0$ and $g_A \to \infty$.

In the KK6 approach, there are two configurations preserving 1/4 of supersymmetry: M2$\subset$KK6 and M5$\subset$KK6. M2 and M5 are thus respectively identified to $m2$ and $m5$, and $l_m = \tilde{L}_p$ where $\tilde{L}_p$ is the eleven dimensional Planck length. The KK6 monopole can be seen as the M-theoretic origin (and thus the strong coupling limit) of the D6-brane. The radius of the NUT direction is thus given by $R_{NUT} = g_A l_s = g_A^{2/3} \tilde{L}_p$. Therefore, the limit above $g_A \to \infty$ becomes $R_{NUT} \to \infty$. Again, in this limit the geometry becomes that of flat space.

It is interesting to note that here as in the former cases of iia and iib theories, the KK monopole description is the more "economic" one, in the sense that one has to take only one limit. However, the other descriptions will be useful to make contact with Matrix theory compactifications.

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In the table below the masses of the different BPS objects of m-theory are listed. Again we consider a compact volume $\tilde{V}_6 = \Sigma_1 \ldots \Sigma_6$.

Brane	Mass
w	$\frac{1}{\Sigma_i}$
$m2$	$\frac{\Sigma_i \Sigma_j}{l_m^3}$
$m5$	$\frac{\tilde{V}_6}{\Sigma_i l_m^6}$

Table 5: mass of the BPS objects in m-theory.

The different ways to obtain m-theory are shown below, along with the relation between the parameters.

	$D6$ $\frac{1}{g_A}, l_s \to 0$	$KK6$ $R_{NUT} \to \infty$
l_m	$g_A^{\frac{1}{3}} l_s$	$\tilde{L}_p$

Table 6: definitions of m-theory parameters.

The duality between m-theory and iia theory can now be made more precise. The relations between the parameters of m-theory compactified on the "7th" direction of radius R_c and iia theory are easily found comparing the tensions of the wrapped and unwrapped $m2$ brane on one side, and of the $f1$ and $d2$ branes on the other side. One finds no surprises:

$$t_a = \frac{R_c}{l_m^3}, \qquad g_a = \left(\frac{R_c}{l_m}\right)^{3/2}.$$

In the KK5(A) and KK6 picture, this is a direct consequence of the relations between M and IIA theories. It is more amusing to see that they indeed correspond to T-duality relations between IIA and IIB when one goes to the D5/D6 picture.

2.4 Relation with Matrix theory compactification

The little theories discussed above are relevant to the description of Matrix theory compactified on higher dimensional tori.

In the original conjecture [4], M-theory in the IMF is described by the Matrix theory of a system of N D0-branes, in the large N limit. The radius R of the compact 11th direction which is used to go to the IMF and the 11 dimensional Planck length l_p enter in the theory of D0-branes via the coupling and the string length of the auxiliary IIA string theory to which the D0-branes belong. If some of the remaining 9 space directions are compactified (on T^d say), one has to correctly include in the Matrix description the additional BPS

states that will fit into representations of the U-duality group of compactified M-theory. A way to achieve this is to take the system of D0-branes on T^d and transform it into a system of N Dd-branes completely wrapped on the dual torus [4, 6]. Then naively one could hope that all the physics of M-theory on T^d would be captured by the SYM theory in $d + 1$ dimensions which is the low-energy effective action of this system of Dd-branes. For completeness we list here the relations between quantities in the string theory in which the Dd-branes live, and Matrix theory variables (see e.g. [15, 32]):

$$ l_s^2 = \frac{l_p^3}{R}, \quad \Sigma_i = \frac{l_p^3}{RL_i}, \quad g_s = \frac{R^{\frac{3-d}{2}}l_p^{\frac{3}{2}(d-1)}}{V_d}, \quad g_{YM}^2 = g_s l_s^{d-3} = \frac{R^{3-d}l_p^{3(d-2)}}{V_d}. \tag{13} $$

l_s and g_s are respectively the string length and coupling; L_i and Σ_i are the sizes of the torus in M-theory and in the auxiliary string theory picture respectively, and $V_d = L_1 \dots L_d$; g_{YM}^2 is the SYM coupling, which is dimensionful in $d \neq 3$. Note that in the end to make contact with M-theory on T^d we have to take the limits $R \to \infty$ and $L_i \to 0$ at fixed l_p, together with the large N limit.

Now for $d \geq 4$ the SYM is ill-defined because non-renormalizable, and thus the SYM prescription for Matrix compactification seems to break down. However, what we should consider as a model for the description of M-theory on a torus is really the "theory on the D-brane" and not only its low-energy field theory limit. Furthermore, to be able to consider a system of N Dd-branes on its own, one has to take a limit in which the bulk physics in the auxiliary string theory decouples. This limit has to be compatible with the other limits discussed in the paragraph above.

For Matrix theory on T^4, it turns out [10] that the theory of D4-branes at strong string coupling coincides with a 6 dimensional (2,0) supersymmetric field theory, which is the theory of N M5-branes in flat space [12]. For Matrix on T^5, the theory of D5-branes at strong coupling is mapped [13] by a IIB S-duality to the theory of N NS5-branes at weak coupling, which is a theory with string-like excitations. Finally, Matrix theory on T^6 is a theory of D6-branes which, at strong coupling, becomes a theory of KK6-monopoles [14, 15]. This 7-dimensional theory has membranes and, as we showed above, has a well-defined structure which has been called m-theory.

We will show in the remainder of this section how all the "phases" of m-theory (i.e. its 7- and 6-dimensional versions) describe M-theory on T^6, and how some particular limits of them yield back the compactifications on T^5 and T^4. In other words, we find the theories mentioned above [10, 13] as limits of the iia and iib little string theories.

Specializing now to $d = 6$, we consider first m-theory in the D6-brane picture. We have for the string coupling:

$$ g_A = \frac{l_p^{15/2}}{R^{3/2}V_6}. \tag{14} $$

For the m-theory to be well-defined, its length scale l_m has to be a fixed parameter. Picking its value from Table 6, it takes the following expression in Matrix theory variables:

$$ l_m^3 = \frac{l_p^{12}}{R^3 V_6}. \tag{15} $$

Note that the limits $V_6 \to 0$ and $R \to \infty$ have to be taken simultaneously and in a definite way, in order to keep a well-defined theory in this limit. Note also that m-theory is valid only in the $g_A \to \infty$ limit, and this is compatible with the above limits since we can re-express $g_A = (l_m/l_p)^3 R$. Also $l_s^2 = l_p^3/R \to 0$ as wanted.

Knowing (15) and the relations between Σ's and L's, we can now translate the masses of the BPS states in m-theory into masses of M-theory objects. We know in advance to which kind of objects they will map to: since the BPS states break half of the supersymmetries of the little theories, they correspond to objects of M-theory in the IMF which break 1/4 of the supersymmetries. These are branes with travelling waves in the 11th direction, i.e. longitudinal branes. The remaining dimensions of these branes are wrapped on the T^6. One could also have deduced this from the fact that the energies of these states will be proportional to n the number of BPS little branes, and independent of N. Since these objects are string-like in the 5 dimensional supergravity to which M-theory is reduced, they should carry the 27 magnetic charges of this theory (i.e. they should fit into the $\overline{\mathbf{27}}$ of the U-duality group $E_6(Z)$ [1]). We indeed find the following identifications: the 15 $m2$ wrapped membranes are mapped to longitudinal M5-branes, the 6 momenta w are mapped to longitudinal M2-branes, and the 6 $m5$ states are longitudinally wrapped KK6 monopoles (the NUT direction being always on the T^6). Their masses can be easily computed from Table 5 and their Matrix couterparts can be found in [32]. All these 27 states can be found also in the iia and iib pictures to be discussed below, although the identification is less straightforward. This clearly convinces that the little string theories are 6-dimensional phases of a description of M-theory on T^6.

We would also like to obtain the spectrum of the 27 electric charges in 5 dimensional supergravity (fitting into the $\mathbf{27}$ of $E_6(Z)$). These correspond to completely wrapped branes in M-theory, or transverse branes in the Matrix theory language (they can be represented as boosted branes). These objects preserve 16 supercharges in the Matrix model, and thus are totally supersymmetric states of the little theory. In the low-energy SYM picture of the little theories, some of these transverse branes can be associated to the electric and magnetic fluxes of the SYM [8, 33]. However the transverse M5-branes are missing from this description, which is thus incomplete (note also that there are no BPS states in the SYM which would represent the longitudinal KK6, or m-theory's $m5$). Going back to the D6-brane picture, one can find all these half-supersymmetric states by embedding in the D6-branes other branes of type IIA theory in a way that they make a non-threshold bound state (the archetype of these states is the supergravity solution of [34]). These states can be found by chains of dualities from [34] and are: F1⊂D6, D4⊂D6 and KK5⊂D6. The energy of these states can also be found in [32]. When there are N D6-branes and n other branes inside them, this energy goes like n^2/N.

We now discuss the other pictures and the other little theories, along with the relations between their parameters and the Matrix theory variables. It is clear that the parameters of the little theories, once expressed in Matrix variables, will no longer depend on the picture by which the little theory was defined. It will be however interesting to check that the limit in which Matrix theory is a good representation of M-theory coincides with the limit in the auxiliary theory in which the little theory is well-defined. As an example, the KK6 picture for m-theory is related to the D6 picture by going from IIA to M on the NUT

direction of the KK6. Then if we call $\tilde{L}_p$ the Planck length of the auxiliary M-theory (not to be confused with the M-theory that we are supposed to describe, characterized by l_p), we have that $\tilde{L}_p = g_A^{1/3} l_s \equiv l_m$ and $R_{NUT} = g_A l_s = (\tilde{L}_p/l_p)^3 R \to \infty$.

The iia theory is most easily obtained going from the D6 to the D5 picture by T-duality. The reason to do this could be that one of the radii of T^6 is much bigger than the others, and we might want to decompactify it eventually. Then the parameters characterizing the IIB auxiliary theory in which the D5 lives are given by:

$$g_B = \frac{l_p^6}{RV_5}, \qquad l_s^2 = \frac{l_p^3}{R}, \qquad R_B = L_6, \tag{16}$$

where $V_5 = L_1 \ldots L_5$. The parameters of the little iia string theory can be easily extracted using Table 2:

$$g_a = \frac{V_5^{1/2}}{l_p^{3/2} L_6}, \qquad t_a = \frac{R^2 V_5}{l_p^9}. \tag{17}$$

The limits of Matrix theory ($L_i \to 0$ and $R \to \infty$) are compatible with keeping g_a and t_a finite. Note however that if $L_6 \to \infty$ instead, then t_a remains fixed while g_a inevitably goes to zero. In this limit all the branes of iia except the $f1$ aquire an infinite tension and thus decouple. We are left with a little string theory at zero coupling, which has exactly the right number of states to describe Matrix theory on T^5. It has indeed 5 winding plus 5 momentum BPS states, which together make up the 10 longitudinal states of Matrix on T^5 [13].

To show that the ii strings tend exactly to the description of Matrix on T^5 given by Seiberg [13], we first go to the NS5(B) picture of iia strings. This is performed by an S-duality, and we obtain for the IIB parameters:

$$\tilde{g}_B = \frac{1}{g_B} = \frac{RV_5}{l_p^6}, \quad \tilde{l}_s^2 = g_B l_s^2 = \frac{l_p^9}{R^2 V_5}, \quad \tilde{R}_B = R_B = L_6. \tag{18}$$

We now see that $\tilde{g}_B = t_a l_p^3/R \to 0$ when $R \to \infty$, and that this limit is independent of L_6. It thus comes out of this picture that the little string theories proposed by Seiberg to describe Matrix on T^5 are the zero coupling limit of the more complete ii little string theories that describe Matrix on T^6.

In order to go to the iib theory, we perform a T-duality along, say, the $\hat{5}$ direction. We obtain a NS5-brane in a IIA theory characterized by:

$$\tilde{g}_A = \frac{\tilde{g}_B \tilde{l}_s}{\Sigma_5} = \frac{RV_4^{1/2} L_5^{3/2}}{l_p^{9/2}}, \quad \tilde{l}_s^2 = \frac{l_p^9}{R^2 V_5}, \quad \tilde{R}_A = L_6, \tag{19}$$

with $V_4 = L_1 \ldots L_4$. It is worth noting that from the iib point of view, the 5th direction has a radius:

$$\Sigma_5' = \frac{\tilde{l}_s^2}{\Sigma_5} = \frac{l_p^6}{RV_4}. \tag{20}$$

This expression has forgotten all dependence on L_5, and thus we should no longer think of the fifth direction of the NS5(A) brane as related to the fifth direction of the original

T^6. Moreover, we can identify $\Sigma'_5 = g^2_{YM(4+1)}$, as in [10]. The parameters of the iib theory are given by:

$$g_b = \frac{L_5}{L_6}, \qquad t_b = \frac{R^2 V_4 L_5}{l_p^9}. \tag{21}$$

Of course, we could have computed this parameters without leaving the little string theories, by t-duality from the iia-theory. For $L_6 \to \infty$ and $L_5 \to 0$, t_b can be fixed but $g_b \to 0$ and we recover the second string theory with 16 supercharges proposed by Seiberg [13]. It is worth noting that $\tilde{g}_A = t_b^{1/2} L_5$ and that the IIA coupling vanishes in this case, but that in the opposite limit, which is appropriate to compactification on T^4, we are at strong coupling. We are thus led to consider the M5 picture of iib strings.

The M5 picture is easily obtained by decompactification of a new direction in the auxiliary M-theory, the radius of which we denote as R_1. The parameters are thus:

$$R_1 = \tilde{g}_A \tilde{l}_s = L_5, \quad R_2 = L_6, \quad L_p^3 = \frac{l_p^9}{R^2 V_4}. \tag{22}$$

If we want the bulk to decouple we have to impose $L_p \to 0$. This combined with $t_b = L_5/L_p^3$ implies that t_b is finite, and we have a little string theory, only if $L_5, L_6 \to 0$. If we want to recover Matrix theory on T^4, we have to take the opposite limit. When $L_5, L_6 \to \infty$, the tension of the little strings becomes very large, only the massless modes contribute, and we are left with a field theory of a special kind, which is however still 6 dimensional. We have thus reproduced the results of [10, 11].

As a last remark on this issue, note that we could have gone to the M5 picture from the D5 one through a T-duality on $\hat{5}$ which would have transformed the D5 into a D4, and then elevating the latter to an M5-brane. Though the labelling of the directions in the auxiliary theory is clearly different in this M5 from the one of the previous paragraph, when expressed in Matrix variables the quantities are exactly the same. This is related to the fact shown in (20) that in the iib picture the "base space" does not refer any more to the original L_5.

3   Theories with 8 supercharges

We propose in this section to define the little string theories with (1,0) supersymmetry in 6 dimensions. Note that this is the highest dimension in which a theory with 8 supercharges can live. We construct the (1,0) theories by analogy with the 10 dimensional relation between $N = 1$ and $N = 2$ string theories.

In 10 dimensions, type I open string theory can be obtained from type IIB string theory [18, 19, 20]. One adds to the IIB theory an $\Omega 9$ orientifold yielding SO open strings [23], and then adds 16 D9-branes to have a vanishing net flux of D9 RR charge. This leads to an $N = 1$ supersymmetric theory with open strings carrying $SO(32)$ Chan-Paton factors. The two heterotic string theories are then obtained by dualities. The $SO(32)$ heterotic theory is found by S-duality from the type I (identifying the D1-brane in the latter to the fundamental heterotic string of the former [20]). The $E_8 \times E_8$ heterotic

theory is obtained by T-duality from the $SO(32)$ one. The $E_8 \times E_8$ theory can also be derived from M-theory compactified on S^1/Z_2 [35].

Our strategy is the following: we define the theories with 8 supercharges using the 5-branes of the ii little string theories, and we then show that the same pattern of dualities as in 10 dimensions arises.

Let us start with the iib little string theory, where we can define a procedure very close to that of [18, 19, 20]. In this theory we have $d5$-branes (cfr. Table 3), which are Dirichlet branes for the little iib fundamental strings, filling the 6-dimensional space-time. They are thus the analogue of the D9-branes of type IIB theory. We now go to one of the precise pictures of section 2.2 to analyze the structure of the theory defined by iib in presence of a certain number n of $d5$-branes.

If we take the KK5(B) picture (see Table 4), the $d5$-brane arises from the $D = 10$ D5-brane with its world-volume inside the KK5. It is now straightforward to identify which BPS states of the iib theory survive the "projection" due to the presence of the $d5$-branes. From the 10-dimensional supersymmetry relations listed in the appendix, we can see that only D1-branes can live at the same time within the KK5 and the D5-branes. The closed $f1$, coinciding with the F1, is no longer a BPS state, and the same occurs to the $d3$ and the $s5$. We are thus left with a theory of open little strings (the open IIB strings within the D5-brane), with a $d1$-brane BPS state. We propose to call this theory type i.

Note that along with the n D5-branes, one can also add an $\Omega5$ orientifold plane[3] without breaking further supersymmetry. Since there are still 3 non-compact transverse directions, the SO or Sp nature of the orientifold and the number of D5-branes is not fixed by simple charge flux arguments. Therefore, unlike the 10 dimensional case, here we can have a priori arbitrary $U(n)$, $SO(2n)$ or $Sp(2n)$ gauge groups on the D5-branes. The $\Omega5$ defines an $\omega5$ little orientifold plane for the iib theory.

If there is only one KK5 brane, the gauge group discussed above corresponds to the gauge group of the little type i string theory. On the other hand, if there are N coinciding KK5 branes (as it should be in a Matrix theory perspective), this issue is more subtle. We return on this at the end of the section.

In order to define a (1,0) closed string theory, we can simply apply the s-duality of section 2.2 to the type i theory. This duality maps the $d1$ branes to the $f1$ little strings, and most notably the $d5$-branes to the $s5$-branes. The only BPS states of this theory are thus the $f1$. We call this theory h_b. We could have directly found this h_b theory from the iib one by piling up n $s5$-branes. If we are allowed to define the s-dual of the $\omega5$ orientifold, then this procedure is reminiscent of the one used by Hull [21] to obtain the heterotic $SO(32)$ theory from type IIB. The possible gauge groups of the h_b theory are the same as the ones for type i theory.

There is still a 5-dimensional object in the little string theories that could be used to define a new (1,0) theory, namely the $s5$-brane of the type iia theory. Taking the KK5(A) picture, we obtain this theory piling up n NS5(A)-branes inside its world-volume. However in this case the gauge symmetry, even in the simplest case of a single KK5, is unclear.

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[3] Much in the same way as it was introduced in [36] in the context of brane configurations describing field theory dualities involving SO and Sp groups.

This is related to the present lack of a definition of a gauge theory associated to the (2,0) theory of n NS5(A)- or M5-branes. We call this little string theory h_a. It is t-dual to the h_b one.

Elevating the picture of a KK5 parallel to NS5-branes in type II theory to M-theory, we find a KK6 with M5 branes defining a domain wall, or boundary of its 7-dimensional world-volume. This is m-theory with $m5$-branes. Thus the h_a theory can be seen as an m-theory compactification in presence of $m5$-branes. This description is very rough and schematic, but could be related to a 7-dimensional analogue of the Horava-Witten mechanism [35] to obtain the $E_8 \times E_8$ heterotic string theory (although in [35] the 9-dimensional objects are really boundaries rather than branes).

We thus see that the pattern of dualities that arises between the theories with 8 and 16 supercharges is very similar to the one between $N = 1$ and $N = 2$ string theories in 10 dimensions. We list in the table below the main characteristics of the (1,0) little string theories.

Theory	Defined by:	BPS objects
i	$ii\mathrm{b} + d5$	$d1$
h_a	$ii\mathrm{a} + s5$	$f1$
h_b	$ii\mathrm{b} + s5$	$f1$

Table 7: main characteristics of the theories with 8 supercharges

We now turn to the discussion of some speculative points related to the theories discussed above.

Consider first the case where the little theories are defined by N branes of the same kind. For simplicity, we specialize to the KK5 picture. In that case, our approach does not help in clarifying which gauge group characterizes the little (1,0) theory. The answer to this problem is however likely to be non-trivial. This can be seen as follows. Take for instance the type i theory. The configuration discussed above to define it involved N KK5 branes parallel to n D5-branes. After a T-duality on the NUT direction we end up with N NS5-branes within n D6-branes. This is related by T-dualities to the configuration studied by Hanany and Witten [37] of D3-branes suspended between NS5-branes. In our case, the direction of the D-branes perpendicular to the NS5-branes is compact (as considered in e.g. [38, 39]). If the NS5 branes were distributed along this compact direction instead of being coincident, the gauge group would have been $U(n)^N$ [38]. In the limit in which the NS5-branes are taken to coincide, it is not clear what gauge theory we get.

The h_a and h_b theories can also be defined by N NS5-branes, with the $s5$-branes provided by n KK5 monopoles (this is obtained by a T-duality from the KK5 picture considered before; here the NS5 and the KK5 branes play the opposite rôle). Since a background of multiple KK5 branes can be related to an ALE space [22], the h_a and h_b theories should be connected to those studied in [40].

Seiberg [13] defines (1,0) little string theories from the world-volume of the 5-branes in the two heterotic string theories. These little theories have however a global $SO(32)$ or $E_8 \times E_8$ symmetry, which is unlikely to arise in our cases. The (1,0) theories of [13] seem thus different from those discussed in this section (in the sense that it should not be possible to derive them from a pure type ii little string framework).

It would be interesting to have an interpretation of the i, h_a and h_b theories in terms

of compactifications of Matrix theory on 6 dimensional manifolds breaking half of the supersymmetries.

As a side remark, it is worth noting that the 5-branes of the little theories play a crucial rôle in the interplay between theories with 16 and 8 supercharges. By analogy, 9-branes in M-theory and in type II theories might be interesting to study. The existence of an M9-brane and NS-like 9-branes of type IIA and IIB theory was indeed discussed in [21].

4   Discussion

We have given in this paper a description of little theories in 6 and 7 dimensions. Our analysis is entirely based on the spectrum of BPS states present in each one of these theories. The focus on BPS states is partly motivated by the application of these little theories to the Matrix theory description of M-theory compactifications, and to the necessity to recover the right U-duality group. If we want to understand more deeply the nature of M-theory, a study of these non-critical string theories and m-theory beyond the BPS analysis is certainly mandatory. A promising avenue is to consider a Matrix approach to these theories, as it was initiated recently in [41, 15, 42, 43, 44, 45] for several related theories.

A full quantum and possibly non-perturbative formulation of these theories will elucidate the relation between the little string theories or m-theory and their low-energy effective action, which must not contain gravity. In other words, this formulation should reproduce the low-energy effective action of the branes used to define the little theories. It may also help in understanding the full structure of the (2,0) field theory in 6 dimensions. An interesting remark is that if we consider the (2,0) and the (1,1) six dimensional field theories as the low-energy effective actions of type iib and iia string theories, then we can observe that both are independent of the (little) string couplings g_b and g_a. This is because the first has no coupling at all, and the second has a SYM coupling $g_{YM}^2 = t_a^{-1}$. This is another characteristic of the 6 dimensional strings which differentiates them from their 10 dimensional sisters.

Acknowledgements

A   Supersymmetry properties and tensions of branes in 10 and 11 dimensions

In this appendix, we give a list of the projections imposed on the supersymmetric parameters of the theory when there is a brane in the background. We also give the tensions of the branes.

In M-theory, we have one Majorana supersymmetric parameter ϵ. The Γ_M matrices are such that the one corresponding to the 11th direction satisfies $\Gamma_{11} = \Gamma_0 \dots \Gamma_9$. We have the following relations (the numbers between brackets indicate the directions longitudinal to the brane, and W[1] stands for a travelling wave or KK momentum in the direction $\hat{1}$):

$$
\begin{aligned}
\text{W[1]:} &\qquad \epsilon = \Gamma_0 \Gamma_1 \epsilon \\
\text{M2[1,2]:} &\qquad \epsilon = \Gamma_0 \Gamma_1 \Gamma_2 \epsilon \\
\text{M5[1..5]:} &\qquad \epsilon = \Gamma_0 \dots \Gamma_5 \epsilon \\
\text{KK6[1..6]:} &\qquad \epsilon = \Gamma_0 \dots \Gamma_6 \epsilon \\
\text{M9[1..9]:} &\qquad \epsilon = \Gamma_0 \dots \Gamma_9 \epsilon
\end{aligned}
\tag{23}
$$

Note that there are no other combinations of the Γ_M matrices which square to the identity. These relations can be obtained from the 11 dimensional supersymmetry algebra including tensorial central charges [3, 21]. Discarding all numerical factors, the tensions of these objects are given as follows. The quantum of mass of a KK momentum on a compact direction of radius R is:

$$
M_W = \frac{1}{R}.
\tag{24}
$$

If L_p is the 11 dimensional Planck length, the tensions of the M2 and M5 branes are:

$$
T_{M2} = \frac{1}{L_p^3}, \qquad T_{M5} = \frac{1}{L_p^6}.
\tag{25}
$$

The tension of a KK6 monopole with a transverse NUT direction of radius R_N is:

$$
T_{KK6} = \frac{R_N^2}{L_p^9}.
\tag{26}
$$

This can be easily obtained from the tension of a D6-brane. We do not discuss here the tension of the M9, which is not used in this paper.

In type II theories, there are 2 Majorana-Weyl spinors ϵ_L and ϵ_R (with reference to the string origin of these susy generators). They satisfy the chirality conditions:

$$
\epsilon_L = \Gamma_{11} \epsilon_L, \qquad \epsilon_R = \eta \Gamma_{11} \epsilon_R,
$$

with $\eta = +1$ for IIB theory and $\eta = -1$ for IIA theory. The supersymmetry projections are the following (we denote by F1 the fundamental strings of each theory):

$$
\begin{aligned}
\text{F1[1]:} &\qquad \left\{ \begin{array}{rcl} \epsilon_L &=& \Gamma_0 \Gamma_1 \epsilon_L \\ \epsilon_R &=& -\Gamma_0 \Gamma_1 \epsilon_R \end{array} \right. \\[4pt]
\text{W[1]:} &\qquad \left\{ \begin{array}{rcl} \epsilon_L &=& \Gamma_0 \Gamma_1 \epsilon_L \\ \epsilon_R &=& \Gamma_0 \Gamma_1 \epsilon_R \end{array} \right. \\[4pt]
\text{NS5[1..5]:} &\qquad \left\{ \begin{array}{rcl} \epsilon_L &=& \Gamma_0 \dots \Gamma_5 \epsilon_L \\ \epsilon_R &=& -\eta \Gamma_0 \dots \Gamma_5 \epsilon_R \end{array} \right. \\[4pt]
\text{KK5[1..5]:} &\qquad \left\{ \begin{array}{rcl} \epsilon_L &=& \Gamma_0 \dots \Gamma_5 \epsilon_L \\ \epsilon_R &=& \eta \Gamma_0 \dots \Gamma_5 \epsilon_R \end{array} \right. \\[4pt]
\text{D}p\text{[1..}p\text{]:} &\qquad \epsilon_L = \Gamma_0 \dots \Gamma_p \epsilon_R
\end{aligned}
\tag{27}
$$

Note that the relations for IIA theory are obtained from those of M-theory compactifying on the 11th direction. Γ_{11} plays thus the rôle of the chiral projector in 10 dimensions, and the supersymmetry parameters are related by $\epsilon_{L(R)} = \frac{1}{2}(1 \pm \Gamma_{11})\epsilon$. Also the relations of IIA and IIB theories are related by T-duality, namely under a T-duality over the $\hat{\imath}$ direction the susy parameters transform (see e.g. [23]) as $\epsilon_L \rightarrow \epsilon_L$ and $\epsilon_R \rightarrow \Gamma_i \epsilon_R$.

The mass of a KK mode W is as in (24). Type II string theories are both characterized by the string length $l_s = \sqrt{\alpha'}$ and by the string coupling constant g. The tension of the fundamental string is:

$$T_{F1} = \frac{1}{l_s^2}. \tag{28}$$

The tension of the solitonic NS5 branes is given by:

$$T_{NS5} = \frac{1}{g^2 l_s^6}. \tag{29}$$

The KK5 monopole has a tension of:

$$T_{KK5} = \frac{R_N^2}{g^2 l_s^8}, \tag{30}$$

where R_N is the radius of the NUT direction. Finally the tensions of the Dp-branes are given by:

$$T_{Dp} = \frac{1}{g l_s^{p+1}}. \tag{31}$$

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